1: Numbering Systems and Conversions -NOTES

TOPIC 1: Numbering Systems

In digital control systems we will need to understand the ways in which these systems represent numbers. We traditionally use the decimal numbering system but computers use binary and when we program and try to understand what the computing system is doing we use hexadecimal and octal systems. The systems work the same but use a different number of characters to represent numbers.

Decimal Numbering System (base 10)

Characters = 0,1,2,3,4,5,6,7,8,9

4	8	7	2	
Thousand's Place	Hundred's Place	Ten's Place	One's Place	= 4x1000 + 8x100 + 7x10 + 2x1

written 4872_d or 4872_{10}

Octal Numbering System (base 8)

Characters = 0,1,2,3,4,5,6,7

4	3	7	
64's place	8's place	1's place	= 4x64+ 3x8 + 7x1

written 437_o or 437_8

Binary Numbering System (base 2)

Characters = 0 and 1

1	1	0	1	
8's place	4's place	2's place	1's place	= 1x8 + 1x4 + 0x2 + 1x1

written 1101_b or 1101_2

Hexadecimal Numbering System (base 16)

Characters = 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

E	4	C	
256's place	16's place	1's place	= 14x256 + 4x16 + 12x1

written $E4C_h$ or $E4C_{16}$

TOPIC 2: Converting Between Numbering Systems
Converting from any numbering system to decimal is almost identical so we will discuss those first.

Binary (base 2) to Decimal (base 10)

Example: Start with 1010110_2

1	0	1	0	1	1	0
2^6	2^5	2^4	2^3	2^2	2^1	2^0

1	0	1	0	1	1	0
64	32	16	8	4	2	1

64+ 0+ 16+0+ 4+ 2 + 0 = 86

Start with the binary number. Below each bit, write the powers of two in increasing order from right to left. Calculate the powers of two and multiply down then add across.

Octal (base 8) to Decimal (base 10)

Example: Start with 864_8

8	6	4
8^2	8^1	8^0

8	6	4
64	8	1

$(8x64)+(6x8)+(4x1) = 564_{10}$

Start with the octal number. Below each digit, write the powers of eight in increasing order from right to left. Calculate the powers of eight and multiply down then add across.

Hexadecimal (base 16) to Decimal (base 10)

Example: Start with $AC5_{16}$

A=10
B=11
C=12
D=13
E=14
F=15

$$\begin{array}{ccc} A & C & 5 \\ 16^2 & 16^1 & 16^0 \\ \hline \\ 10 & 12 & 5 \\ 256 & 16 & 1 \\ \hline \end{array}$$

Start with the hex number. Below each digit, write the powers of 16 in increasing order from right to left. Calculate the powers of 16 and multiply down then add across.

$(10x256)+(12x16)+(5x1) = 2757_{10}$

Decimal (base 10) to Binary (base 2)

Example: Start with 46_{10}

$$\frac{46}{2} = 23\,R0 \qquad \frac{23}{2} = 11\,R1$$

$$\frac{11}{2} = 5\,R1 \qquad \frac{5}{2} = 2\,R1$$

$$\frac{2}{2} = 1\,R0 \qquad \frac{1}{2} = 0\,R1$$

Concatenate the remainders and the result is 101110

Therefore $46_{10} = 101110_2$

Decimal (base 10) to Octal (base 8)

Example: Start with 964_{10}

$$\frac{964}{8} = 120\text{ R}4 \qquad \frac{120}{8} = 15\text{ R}0$$

$$\frac{15}{8} = 1\text{R }7 \qquad \frac{1}{8} = 0\text{R }1$$

Concatenate the remainders and the result is 1704

Therefore $964_{10} = 1704_{8}$

Decimal (base 10) to Hexadecimal (base 16)

Example: Start with 8294_{10}

$$\frac{8294}{16} = 518\text{ R}6 \qquad \frac{518}{16} = 32\text{ R}6$$

$$\frac{32}{16} = 2\text{ R}0 \qquad \frac{2}{16} = 0\text{R }2$$

Concatenate the remainders and the result is 2066

Therefore $8294_{10} = 2066_{16}$

Hexadecimal (base 16) to Binary (base 2) and reverse

Example: Start with $CE45_{16}$

C	E	4	5
1100	1110	0100	0101

$110011100100010 1_2$

Example: Start with 010111110000101_2

(0)010	1111	1000	0101
2	F	8	5

$2F85_{16}$

Octal (base 8) to Binary (base 2) and reverse

Example: Start with 3742_8

3	7	4	2
011	111	100	010

011111100010_2

Example: Start with 10010011111101_2

(0)10	010	011	111	101
2	2	3	7	5

22375_8

Octal to Hexadecimal and reverse

Example: Start with 3756_8

3	7	5	6_8
011	111	101	110

0111	1110	1110
7	E	E_{16}

Convert the octal to binary then convert the binary to hex. The reverse is just the opposite procedure.